

# Regular Sparse Arrays: the Impact of Grating Lobes on Radio Astronomical Observations

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### **Abstract**

Large radio astronomy array systems can potentially be realized cost effectively when built with sparse-regular antenna arrays, a concept which is traditionally avoided due to the presence of grating lobes. This paper discusses the impact of grating lobes on the system performance of an aperture array radio telescope.

### 1. Introduction

The radio astronomy community realized in the mid-1990s that the next generation radio telescopes would have to provide significantly more collecting area than the then-current telescopes in order to satisfy the needs of the radio astronomy community. This led to the conception of the Square Kilometre Array (SKA) [1,2]: a telescope with a collecting area of a square kilometre. Besides dishes, Aperture Arrays (AA) will be used for the lower frequencies, up to 350MHz, were dishes are impractical. AAs are also under consideration for the mid-frequency regime (350-1500 MHz).

SKA radio telescopes require: 1) high sensitivity, determined by system noise temperature and collecting area; 2) high resolution, demanding sufficiently long baseline; 3) high survey speed, requiring a large Field of View (FoV); 4) high dynamic range; and 5) large frequency bandwidths.

Implementing 1-3 is straightforward with an AA system, in particular with the very high FoV capability of AAs. However, achieving a high dynamic range requires careful consideration. For SKA2 an imaging dynamic range of 70dB is required.

## 2. System

Sparse regular arrays are an interesting solution for large radio astronomy aperture (phased) arrays [3]. Whilst the advantages in cost and processing power of regular arrays might be obvious, the appearance of grating lobes is normally considered a very serious limitation. In a radar system, target location is crippled when grating lobes create alternative target options. But operating sparse dish arrays is well understood in radio astronomy; in these systems, the intrinsic dish gain reduces the effect of farout grating lobes, even when operated in a (phased-up) tied array mode. The Westerbork Radio Synthesis Telescope (WSRT) is a nice example of a regular sparse dish array: 12 of the 14 dishes are placed at an interval of 144m on an East-West line. For HI frequencies (1420MHz) this means a separation d=686 λ. Operating WSRT in tied array mode many grating lobes are produced, a number of them inside the main beam of the dish. This is interesting for, e.g., pulsar search science, when positioning of (new) sources is not directly required. For high dynamic range imaging, grating lobes are however undesirable and avoided in the correlation mode. An operational AA system with a sparse regular structure is LOFAR [4].

AAs are under consideration for the frequencies up to 1500MHz in order to benefit from the advantages of AAs when compared with dishes: 1) multiple beams, 2) large FoV, 3) low cost antennas. Table 1 gives the main parameters of a typical AA station option. 150 of these stations will fulfill the SKA2 requirements. The proposed sparse design has 4 to 6 times less elements when compared with a dense solution. The impact of the resulting grating lobes will be discussed in the next section.

Table 1 Typical AA station parameters

| Parameter                  |                      |
|----------------------------|----------------------|
| Station diameter (D)       | 45 m                 |
| Number of antenna elements | 18.000               |
| Configuration              | Regular, circular    |
| Antenna spacing            | λ/2 at 500MHz: 0.3 m |
| Operating frequency range  | 350-1500 MHz         |



Figure 1. Artist impression of an AA system.

# 3. Impact of grating lobes

In [3] it is proposed to consider station rotation as a technique to reduce the impact of grating lobes when station beams are correlated. LOFAR [4] successfully explored this as well. Frequency and time effects will further reduce the impact of grating lobes. The following sections will give a qualitative assessment of these two effects.

# 3.1 Frequency effects

In order to estimate the effect of grating lobes for wide bandwidth observations, let us consider a linear array, with N elements and regular element spacing of d. The normalized radiation pattern of the array is given by [5]:

$$G(\theta) = \frac{\sin^2[N\pi\left(\frac{d}{\lambda}\right)(\sin\theta - \sin\theta_d)]}{N^2\sin^2[\pi\left(\frac{d}{\lambda}\right)(\sin\theta - \sin\theta_d)]}$$
(1)

Here,  $\lambda$  is the nominal wavelength and  $\theta_d$  the direction of the beam as a result of uniform beam steering by means of phase or time delay beam forming. The grating lobes appear at an angle of  $\theta_g$  whenever the denominator is zero, or when:

$$\left|\sin\theta_g - \sin\theta_d\right| = \pm n\frac{\lambda}{d} \tag{2}$$

From (2) it is clear that the position of the grating lobe not only depends on the direction of the main beam but also on the wavelength. Energy of (strong) sources in the grating lobe will be reduced in wide frequency bandwidth observations due to the wavelength dependency of the grating lobe pointing angle. In order to assess this effect, we consider the first grating, for n=1:

$$\theta_g = \arcsin\left(\frac{\lambda}{d} + \sin\theta_d\right) \tag{3}$$

For two wavelengths,  $\lambda$  and  $\lambda$ ', with  $\theta_g$  and  $\theta_g$ ' respectively, we set the difference between  $\theta_g$  and  $\theta_g$ equal to the beam width:  $\theta_g - \theta_g{'} = \theta_B$ 

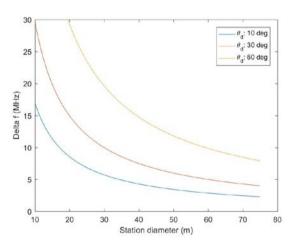
$$\theta_g - \theta_g' = \theta_B \tag{4}$$

Using (3) and (4) the wavelength at which the grating is moved by a beam width can be calculated:

$$\lambda' = -d[\sin(\theta_q - \theta_B) - \sin\theta_d] \tag{5}$$

The half power beam width of the grating lobe is identical to the main (desired) beam:  $\theta_B = \lambda/(d(N-1))$  in zenith or  $\lambda/(d(N-1)\cos\theta_d)$  for off zenith pointing..

In figure 2. the difference in frequency between  $\lambda$  and  $\lambda$ ' is plotted for three scan angles at 1GHz and using a station diameter D=0.3N. For a typical situation using the 45-m diameter station proposed in table 1, the equivalent linear array has N=150.



**Figure 2.** Indication of grating lobe channel pollution for a  $\lambda/d=1$  regular array

From figure 2 we can see that for modest scan angles and a station size in the order of 45-m diameter or more the grating lobe has moved in less than 10MHz. This means that adjacent 10MHz frequency bins are sufficiently different in terms of grating energy for sources in the grating lobes smaller than the beam size.

### 3.2 Time effects

Repointing of the array, required to keep-up with earth rotation, will reduce the effect of grating lobes since the repointing will move grating lobs to another location on the sky as the main lob will continue to follow the target source.

The repointing depends on:

Location of the array. The South African SKA site is at a latitude of 30° south.

- Position of the target source with respect to the array, the declination δ
- The size of the array

Earth rotation (ER) equals to  $\frac{2\pi}{24 \ hours} = 0.0727 \ mrad/s$  or  $0.0042^{\circ}/s$ . For sources above the equator a nominal repointing time can be set to:

$$t = \frac{\theta_B}{ER} \tag{6}$$

The beam width,  $\theta_B$ , widens to  $\mathcal{N}(\cos(\theta_d)D_{stat})$  with  $\theta_d$  the zenith angle. In Figure 3. the repointing time is plotted for 1GHz and d=0.3 for three different pointing angles. Practical situations are somewhat more complex. E.g. at high latitude (pole) sources do not need repointing. Still, earth rotation will move the grating lobe on the sky. For a maximum scan angle of  $60^\circ$  the beam doubles, increasing the time the grating lob will receive significant energy from a potential strong source to nearly 200 s for a 45-m station.

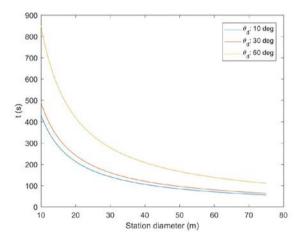


Figure 3. Nominal repointing time

Typical image processing data is handled in 10-s time slots. Strong sources in the grating lobes will therefore affect multiple time slots.

# 4. Conclusion

The qualitative effect of grating lobes on radio astronomy observations with respect to frequency and time dependency has been assessed for typical SKA aperture arrays. The impact of grating lobes will be strongly reduced when stations are rotated and station beams correlated, as argued in [2]. Further reduction is achieved, in the case of wide bandwidth science and longer integrations will mitigate the negative effect. However, for a typical station size the additional suppression due to time effects is limited: multiple time slots will be affected when strong signals are received in the gratings lobes. Since grating lobes in a regular array are deterministic certain station beams could be excluded for processing, for example when the grating lobe is pointed at the sun or another know strong sources. This will however reduce the sensitivity of the system.

## 5. References

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