Coupling Random Matrix approach and Wigner function technique to study the directivity of the EM field emitted by an open chaotic reverberation chamber.

J.-B. Gros*(1), P. del Hougne(1), G. Gradoni(2) and G. Lerosey(1)
(1) Institut Langevin, CNRS UMR 7587, ESPCI Paris, PSL Research University, 75005 Paris, France
(2) School of Mathematical Sciences and George Green Institute for Electromagnetics Research, University of Nottingham, University Park, Nottingham, NG7 2RD, UK

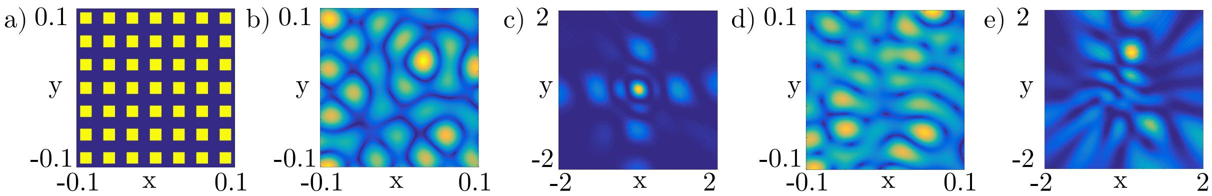

Figure 1. a) : Mask $M(\vec{r})$ corresponding to the grating. $M = 1$ (yellow surfaces), $M = 0$ (blue surface). b) and d) : Near field obtained for two different RMT simulations. c) and e) : far fields corresponding respectively to b) and d) after the mask a) has been applied to them.

In this paper, we propose to investigate the directivity of the electromagnetic (EM) field emitted by an open cavity [1]. Because of its universal statistical behaviour, which is fully capture by the Random Matrix Theory (RMT), it is interesting to assume that the cavity is a chaotic reverberation chamber [2]. In recent years, such cavities have been involved in a variety of applications ranging from electromagnetic compatibility [2] to microwave imaging [3] or telecommunication and energy harvesting. [4]. Experimentally, an open chaotic cavity can be designed by replacing parts of its walls by a subwavelength grating (see [1]). In the present study, we will emulate the response of such cavities via RMT simulations. Assuming an excitation by an elementary oscillating dipole located at $\vec{r}_0$ at frequency f_0, the response of the cavity is given by the dyadic Green's Tensor (DGT) $\overline{\overline{G}}(\vec{r},\vec{r}_0,f_0)$. Far from the source region, the DGF can be expanded over the resonances [2] :

$$\overline{\overline{G}}(\vec{r},\vec{r}_0,f_0) = \sum_{n=1}^{\infty} \frac{\vec{E}_n(\vec{r})\vec{E}_n(\vec{r}_0)}{(\tilde{k}_n(1-(1+i)/2Q_n))^2 - k_0^2}, \text{ with } k_0 = 2\pi f_0/c. \tag{1}$$

In our RMT approach, the eigenvalues $\tilde{k}_n$ are computed from rescaled eigenvalues of an adequate random matrix [2] and all modal quality factors Q_n are replaced by a mean quality factor Q (which is a parameter of our study). Each eigenfield $\vec{E}_n(\vec{r})$ is a "synthetic" eigenfield constructed by using a random superposition of plane waves where the norm of their wave vector is fixed to $\tilde{k}_n$ and $\vec{r}$ is a position vector in the plane $z = 0$ where we apply a mask $M(\vec{r})$ corresponding to the grating. Finally, we have computed the far field of $M\overline{\overline{G}}$, for two different RMT simulations where $f_0 = 5$GHz and Q=700. We observe that the far field of an open chaotic cavity could be directive (Fig. 1.c)) or not (Fig. 1.e)). Looking only at the spatial distribution of a near field (Fig. 1.b) and 1d)) seems to be insufficient to predict the directivity of its far field. In order to understand the origin of the difference of behaviour shown in Fig. 1, we will study a quantity derived from the spatial two point correlation function, $\Gamma_0(\vec{r}_1,\vec{r}_2) = \langle M\overline{\overline{G}}(\vec{r},\vec{r}_1)M\overline{\overline{G}}(\vec{r},\vec{r}_2)\rangle_{\vec{r}}$, the so-called *Wigner function* [5]. This function is a phase space representation and therefore it contains both positional and directional information of the EM field. It has been applied successfully to characterize the radiation of planar EM sources.

References

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