



UAPO Solution for the Plane Wave Diffraction by a 90° Coated Wedge

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Abstract

This paper provides an alternative solution to the diffraction problem associated to a plane wave incident on a perfectly conducting wedge coated by dielectric layers. The skew incidence with respect to the edge of a right-angled structure is considered. The proposed analytical approach starts by considering the scattering integral and using a physical optics approximation of the electric and magnetic equivalent surface currents located on the external faces of the coated wedge. Useful approximations and analytical calculations allow one to apply a uniform asymptotic evaluation to the resulting integrals to obtain the diffraction coefficients. They are easy to handle and able to compensate the geometrical optics discontinuities.

1. Introduction

The canonical diffraction problem tackled in this manuscript refers to a uniform plane wave impacting on a perfectly conducting (PEC) wedge coated by dielectric layers. Such a problem is well known in literature and many methods have been proposed to solve it for a limited number of cases. The interest is justified by its occurrence in many application areas such as radar detection and wave propagation analysis in man-made environments.

Uniform Asymptotic Physical Optics (UAPO) solutions have been determined by the authors to compute the high-frequency diffraction coefficients associated to penetrable and impenetrable wedges [1]-[5]. They can be used in the Uniform Theory of Diffraction (UTD) [6] framework to compensate the discontinuities at the Geometrical Optics (GO) boundaries. They result from an analytic procedure and possess the ease of use, which is typical of the heuristic solutions. However, they are approximate and furnish accurate results save for the case of grazing incidence and in correspondence of the interfaces. According to the above statements, the UAPO solutions can attract the interest of the electromagnetic engineers since they are reliable and user-friendly. The characteristics of the UAPO solutions suggest exploiting the same procedure for solving the considered diffraction problem.

2. UAPO Diffracted Field

A linearly polarized plane wave impact on a right-angled perfectly conducting wedge with a uniform, isotropic and homogeneous layer coating on both faces. The material is non-magnetic and its relative permittivity is $\epsilon_r = \epsilon' - j\epsilon''$. The incidence and diffraction directions are indicated by (β', ϕ') and (β, ϕ) , respectively, and s is the distance from the diffraction point to the observation point P (see Fig. 1).

The target is to determine the closed form expression of the UAPO matrix \underline{D} to evaluate the diffracted electric field \underline{E}^d from the knowledge of the incident field \underline{E}^i , i.e.,

$$\underline{E}^d = \begin{pmatrix} E_\beta^d \\ E_\phi^d \end{pmatrix} = \underline{D} \frac{\exp(-jk_0 s)}{\sqrt{s}} \begin{pmatrix} E_{\beta'}^i \\ E_{\phi'}^i \end{pmatrix} = \underline{D} \frac{\exp(-jk_0 s)}{\sqrt{s}} \underline{E}^i \quad (1)$$

where k_0 is the free-space propagation constant.

In the PO approximation, the incident field defines the electric (\underline{J}_s) and magnetic (\underline{J}_{ms}) surface currents on the external surfaces S_0 and $S_{3\pi/2}$ of the coated wedge. Such sources can be considered in the radiation integral to evaluate the scattered electric field \underline{E}^s in the far-field approximation, i.e.,

$$\underline{E}^s \equiv -jk_0 \iint_S \left[(\underline{I} - \hat{R}\hat{R}) \zeta_0 \underline{J}_s + \underline{J}_{ms} \times \hat{R} \right] G(\underline{R}) dS \quad (2)$$

where $S = S_0 \cup S_{3\pi/2}$, ζ_0 is the free-space impedance, \hat{R} is the unit vector from the source point at \underline{r}' to $P(\underline{r})$ and $R = |\underline{r} - \underline{r}'|$. The symbol \underline{I} denotes the identity matrix and $G(\underline{R}) = \exp(-jk_0 R)/(4\pi R)$. Since the \underline{J}_s and \underline{J}_{ms} phase functions coincide on the considered surface, f.i.,

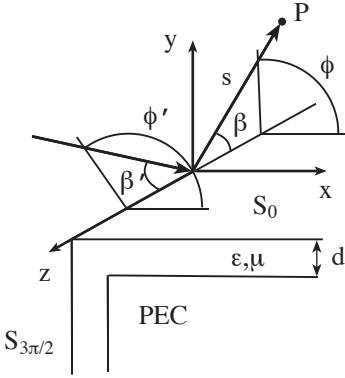


Figure 1. Geometry of the problem.

$\underline{J}_s = \underline{J}'_s \exp(\phi_0(r'))$ and $\underline{J}_{ms} = \underline{J}'_{ms} \exp(\phi_0(r'))$ on S_0 , it results:

$$\begin{aligned} \underline{E}^s &= \begin{pmatrix} E_\beta^s \\ E_\phi^s \end{pmatrix} \equiv U_0 \left[(\underline{\underline{I}} - \hat{s}\hat{s}) \zeta_0 (\underline{J}'_s)_{S_0} + (\underline{J}'_{ms})_{S_0} \times \hat{s} \right] I_0^s + \\ &\quad U_{3\pi/2} \left[(\underline{\underline{I}} - \hat{s}\hat{s}) \zeta_0 (\underline{J}'_s)_{S_{3\pi/2}} + (\underline{J}'_{ms})_{S_{3\pi/2}} \times \hat{s} \right] I_{3\pi/2}^s = \\ &= \left(U_0 \underline{\underline{M}}_0 I_0^s + U_{3\pi/2} \underline{\underline{M}}_{3\pi/2} I_{3\pi/2}^s \right) \begin{pmatrix} E_{\beta'}^i \\ E_{\phi'}^i \end{pmatrix} \end{aligned} \quad (3)$$

where the unit vector \hat{s} indicates the diffraction direction on the Keller's cone, $U_0 = 1$ if $0 < \phi' < \pi$ and $U_0 = 0$ otherwise, $U_{3\pi/2} = 1$ if $\pi/2 < \phi' < 3\pi/2$ and $U_{3\pi/2} = 0$ otherwise. Note that the approximation $\hat{R} \equiv \hat{s}$ is adopted in the square brackets of the integrand in (2) without affecting the expression of R . The resulting integrals

$$I_0^s = -\frac{jk_0}{4\pi} \iint_{S_0} \frac{\exp(\phi_0 - j\beta_0 R)}{R} dS_0 \quad (4)$$

$$I_{3\pi/2}^s = -\frac{jk_0}{4\pi} \iint_{S_{3\pi/2}} \frac{\exp(\phi_{3\pi/2} - j\beta_0 R)}{R} dS_{3\pi/2} \quad (5)$$

are reduced to the form of a typical integral, which is evaluated by using the steepest descent method in the high-frequency approximation. Accordingly, the UAPO diffraction contributions I_0^d and $I_{3\pi/2}^d$ extracted from I_0^s and $I_{3\pi/2}^s$ are given by:

$$I_0^d = \frac{\exp(-j\pi/4)}{2\sqrt{2\pi k_0}} F(\beta', \phi', s, \phi) \frac{\exp(-jk_0 s)}{\sqrt{s}} \quad (6)$$

$$I_{3\pi/2}^d = \frac{\exp(-j\pi/4)}{2\sqrt{2\pi k_0}} F\left(\beta', \frac{3}{2}\pi - \phi', s, \frac{3}{2}\pi - \phi\right) \frac{\exp(-jk_0 s)}{\sqrt{s}} \quad (7)$$

where

$$F_t\left(\beta', \phi', s, \phi\right) = \frac{F_t\left(2k_0 s \sin^2 \beta' \cos^2 \left(\frac{\phi \pm \phi'}{2}\right)\right)}{\sin^2 \beta' (\cos \phi + \cos \phi')} \quad (8)$$

if $F_t(\cdot)$ denotes the UTD transition function [6]. Note that the sign + (−) must be used when P is located in the half-space over (under) the involved surface. The UAPO diffracted field is then expressed as

$$\begin{pmatrix} E_\beta^d \\ E_\phi^d \end{pmatrix} = \left(U_0 \underline{\underline{M}}_0 I_0^d + U_{3\pi/2} \underline{\underline{M}}_{3\pi/2} I_{3\pi/2}^d \right) \begin{pmatrix} E_{\beta'}^i \\ E_{\phi'}^i \end{pmatrix} \quad (9)$$

The corresponding matrix $\underline{\underline{D}}$ is determined by comparing (1) and (9), thus obtaining the following expression:

$$\underline{\underline{D}} = \left(U_0 \underline{\underline{M}}_0 I_0^d + U_{3\pi/2} \underline{\underline{M}}_{3\pi/2} I_{3\pi/2}^d \right) \frac{\sqrt{s}}{\exp(-jk_0 s)} \quad (10)$$

As evident, the matrices $\underline{\underline{M}}_0$ and $\underline{\underline{M}}_{3\pi/2}$ play an important role to evaluate $\underline{\underline{D}}$ and therefore it is important to spend effort to characterize them in the following.

The expressions of \underline{J}'_s and \underline{J}'_{ms} are the core of $\underline{\underline{M}}_0$ and $\underline{\underline{M}}_{3\pi/2}$. In particular, \underline{J}'_s and \underline{J}'_{ms} are formulated in terms of the incident field components and the reflection coefficients Γ for parallel and perpendicular polarizations, f.i.,

$$\begin{aligned} (\underline{J}'_s)_{S_0} &= \frac{1}{\zeta_0} \left[(1 - \Gamma_\perp) \sin \beta' \sin \phi' E_\perp^i \hat{u}_\perp + \right. \\ &\quad \left. + (1 + \Gamma_\parallel) E_\parallel^i (\hat{y} \times \hat{u}_\perp) \right] \end{aligned} \quad (11)$$

$$\begin{aligned} (\underline{J}'_{ms})_{S_0} &= \left[(1 - \Gamma_\parallel) \sin \beta' \sin \phi' E_\parallel^i \hat{u}_\perp + \right. \\ &\quad \left. - (1 + \Gamma_\perp) E_\perp^i (\hat{y} \times \hat{u}_\perp) \right] \end{aligned} \quad (12)$$

where \hat{u}_\perp is the unit vector perpendicular to the standard incidence plane. The coefficients $\Gamma_{\parallel, \perp}$ are evaluated by considering the Equivalent Transmission Line (ETL) model for a free-space plane wave incident on a lossy dielectric layer with PEC backing.

Accounting for the transformation matrices from the incident ray-fixed coordinate system to the diffraction one, it results:

$$\underline{\underline{M}}_0 = \underline{\underline{M}}_1 [\underline{\underline{M}}_2 \underline{\underline{M}}_4 \underline{\underline{M}}_5 + \underline{\underline{M}}_3 \underline{\underline{M}}_4 \underline{\underline{M}}_6] \underline{\underline{M}}_7 \quad (13)$$

with

$$\underline{\underline{M}}_1 = \begin{pmatrix} \cos\beta'\cos\phi & \cos\beta'\sin\phi & -\sin\beta' \\ -\sin\phi & \cos\phi & 0 \end{pmatrix} \quad (14)$$

$$\underline{\underline{M}}_2 = \begin{pmatrix} 1-\sin^2\beta'\cos^2\phi & -\cos\beta'\sin\beta'\cos\phi \\ -\sin^2\beta'\sin\phi\cos\phi & -\cos\beta'\sin\beta'\sin\phi \\ -\cos\beta'\sin\beta'\cos\phi & \sin^2\beta' \end{pmatrix} \quad (15)$$

$$\underline{\underline{M}}_3 = \begin{pmatrix} 0 & -\sin\beta'\sin\phi \\ -\cos\beta' & \sin\beta'\cos\phi \\ \sin\beta'\sin\phi & 0 \end{pmatrix} \quad (16)$$

$$\underline{\underline{M}}_4 = \frac{1}{\sqrt{1-\sin^2\beta'\sin^2\phi'}} \begin{pmatrix} -\cos\beta' & -\sin\beta'\cos\phi' \\ -\sin\beta'\cos\phi' & \cos\beta' \end{pmatrix} \quad (17)$$

$$\underline{\underline{M}}_5 = \begin{pmatrix} 0 & (1-\Gamma_{\perp})\sin\beta'\sin\phi' \\ 1+\Gamma_{\parallel} & 0 \end{pmatrix} \quad (18)$$

$$\underline{\underline{M}}_6 = \begin{pmatrix} (1-\Gamma_{\parallel})\sin\beta'\sin\phi' & 0 \\ 0 & -(1+\Gamma_{\perp}) \end{pmatrix} \quad (19)$$

$$\underline{\underline{M}}_7 = \frac{1}{\sqrt{1-\sin^2\beta'\sin^2\phi'}} \begin{pmatrix} \cos\beta'\sin\phi' & \cos\phi' \\ -\cos\phi' & \cos\beta'\sin\phi' \end{pmatrix} \quad (20)$$

The matrix $\underline{\underline{M}}_{3\pi/2}$ has the same formulation of $\underline{\underline{M}}_0$ save for considering proper reflection coefficients and involved angles.

3. Numerical tests

Numerical simulations have been performed to test the performance of the proposed UAPO solution. A set of results relevant to the case of normal incidence ($\beta'=90^\circ$) follows. The incident field is assumed to have $E_{\beta'}^i=1$,

$E_{\phi'}^i=0$ and the observation domain is a circular path having radius $\rho=5\lambda_0$, where λ_0 is the free-space

wavelength. The dielectric layer is characterized by thickness $d=0.2\lambda_0$ and relative permittivity $\epsilon_r=5-j0.005$.

The magnitudes of the GO and UAPO diffracted fields are shown in Fig. 2 when $\phi'=45^\circ$. This incidence direction implies $U_0=1$ and $U_{3\pi/2}=0$, so that only S_0 contributes to the evaluation of the UAPO diffracted field. As expected, the GO field curve possesses two discontinuities in correspondence of the reflection and incident shadow boundaries at $\phi=135^\circ$ and $\phi=225^\circ$, and the UAPO diffracted field guarantees the continuity of the total field across them as shown in Fig. 3. Figures 4 and 5 refer to $\phi'=150^\circ$, so that $U_0=1$ and $U_{3\pi/2}=1$. The UAPO diffracted field works well also in this case by assuring the continuity of the total field across the GO shadow boundaries. Such a required performance is evident also in last figures, which refer to $\phi'=220^\circ$ and then only the contribution of $S_{3\pi/2}$ exists.

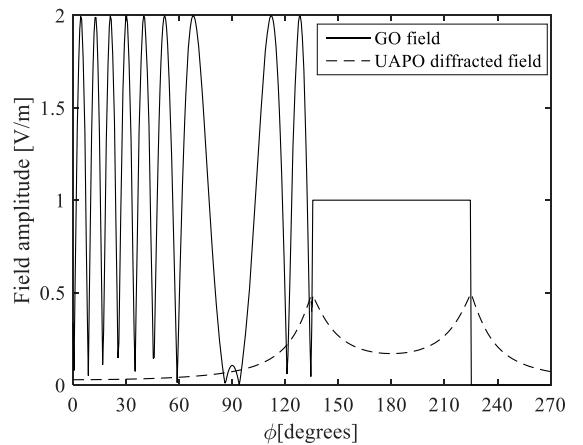


Figure 2. GO and UAPO diffracted fields if $\phi'=45^\circ$.

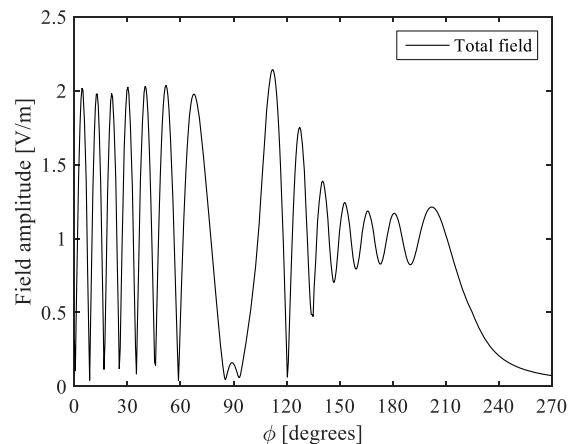


Figure 3. Total field if $\phi'=45^\circ$.

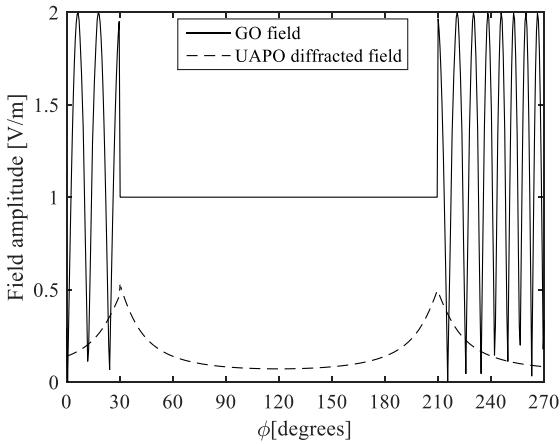


Figure 4. GO and UAPO diffracted fields if $\phi' = 150^\circ$.

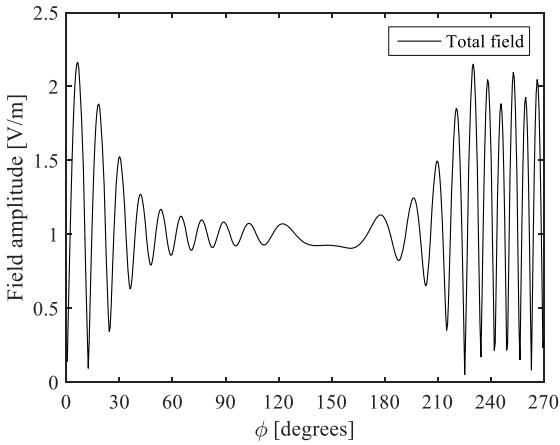


Figure 5. Total field if $\phi' = 150^\circ$.

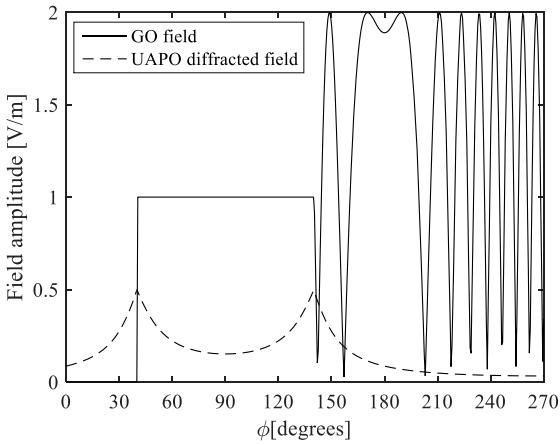


Figure 6. GO and UAPO diffracted fields if $\phi' = 220^\circ$.

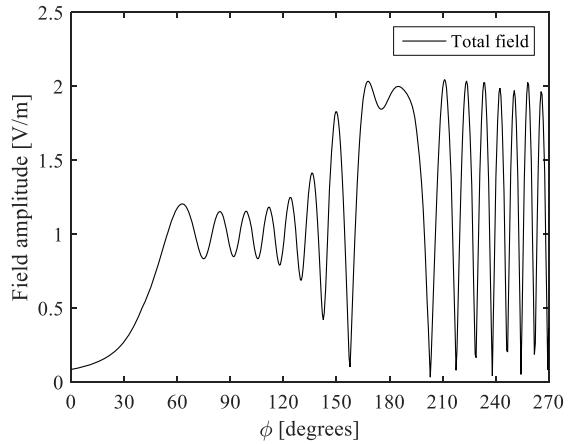


Figure 7. Total field if $\phi' = 220^\circ$.

7. References

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